Music and Mathematics in Latin America: Major Developments in the Last 25 Years

Gabriel Pareyon (CENIDIM-INBA, Mexico City & CUCSH-UDG, Guadalajara)
ORCID: 0000-0002-3471-0102 gabrielpareyon@gmail.com

Carlos Almada (Universidade Federal do Rio de Janeiro)
ORCID: 0000-0001-5888-2868 carlosalmada@musica.ufrj.br

Carlos Mathias (Universidade Federal Fluminense)
ORCID: 0000-0001-7949-2324 profcarlosmathias@gmail.com

Cecilia Saraiva (Universidade Federal do Estado do Rio de Janeiro)
ORCID: 0000-0002-0285-4494 cecilia.saraiva@uniriotec.br

Daniel Moreira
Universidade Federal do Rio de Janeiro & Conservatório Brasileiro de Música
ORCID: 0000-0001-7964-4703 danielspro@hotmail.com

Hugo Carvalho (Universidade Federal do Rio de Janeiro)
ORCID: 0000-0003-0776-0400 hugo@dme.ufrj.br

Liduino Pitombeira (Universidade Federal do Rio de Janeiro)
ORCID: 0000-0002-1141-655X pitombeira@musica.ufrj.br

Pauxy Gentil-Nunes (Universidade Federal do Rio de Janeiro)
ORCID: 0000-0003-1548-7694 pauxy.contact@gmail.com

Bruno Mesz
Instituto de Investigación en Arte y Cultura & Universidad Nacional de Tres de Febrero
ORCID: 0000-0002-4941-818X bruno.mesz@gmail.com

Pablo Amster (Universidad de Buenos Aires & IMAS-CONICET)
ORCID: 0000-0003-2829-7072 pamster@dm.uba.ar

Pablo Riera (Universidad Nacional de Quilmes)
ORCID: 0000-0001-8570-4688 pr@lapso.org
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Abstract

This text is an overview for Latin America across the field of music and musicology intersecting mathematics, including the advances from perspectives of experimentation, creation, analysis and pedagogy throughout interdisciplinary developments, particularly in the field of computational science and philosophy of science. Our main goal is to spread worldwide the richness and variety of research on music and mathematics in Latin America as well as stimulate further investigation in this fascinating intersection.

***Keywords**: Music and Mathematics. Latin-American Research. Survey.*

I. Introduction

The first text on this issue presents a compact and stimulating historical overview on the diversity of concepts and methods of the native people of Latin America on connections between music, mathematics, and the natural sciences. Despite the breakage of this evolutionary line due to the colonization and the consequent loss of information, the cultural richness of our subcontinent is element of study by scholars worldwide. This text is the result of a joint effort in collecting and summarizing some of the major developments made on research on music and mathematics in Latin America —at least on the last 25 years—, as means of providing an initial step on unifying and displaying our cultural and scientific richness on these subjects. By no means this text is exhaustive, nor it intend to be;[1] however its main importance lies on the fact that it is the very first document that broadly catalogues this kind of research in Latin America.

In order to facilitate the reading, the countries and/or regions are divided among the sections and subsections of this text, in alphabetical order. At the very end an extensive list of bibliographical references are provided, splitted in the same fashion as the rest of the text, but with continuous numeration. This work would not be possible without the collaboration of Rodolfo Coelho de Souza, Paulo de Tarso Salles, Marco da Silva Sampaio, Pedro Kroeger, Didier Guigue, Jônatas Manzolli and Luiz Biscainho, which provided us with information on their respective groups and research and to whom we thank a lot.

Finally, we think that the *MusMat • Brazilian Journal of Music and Mathematics* has a paramount responsibility on hosting not only this first overview but also on encouraging fellow researchers from Latin America on gathering and spreading similar information. In order to stimulate this, from now on the MusMat Journal will dedicate a section to publish information with similar content. Therefore, if your research group, region, or country is missing, please feel free to submit an analogous document to the Journal.

II. Argentina and Chile

Interest in the relations between mathematics and music has developed in the last fifteen years in Argentina. Musicological works have been published using statistical methods, complex networks, group theory, combinatorics, differential equations, signal analysis, deep neural networks and other computational methods. Here we present a brief overview of these investigations.

Physicist Damián Zanette works in statistical analysis of music and musical performance. In his paper "Zipf's law and the creation of musical context" [25] he considers the emergence of musical context from compositional decisions, privileging some elements over others. He shows that the assumption that the frequency of use of a musical "word" (a note with its associated duration in this case) increases proportionally to the number of its previous occurrences in the piece leads to a Zipf law for the vocabulary. A tonal context such as that of Bach's or Mozart's works is more consistent than Schoenberg's free atonality, and that is reflected in Zipf exponents: small exponents indicate a compact lexicon and, consequently, a robust, stable and well-defined context, while larger exponents reveal an abundant vocabulary.

In another article, Zanette and the composer and musicologist Fernando Benadon study microtiming in 48 performances of J.S. Bach's C Major Prelude from The Well Tempered Clavier, Book I, defining a measure of "rubato depth" and linking it with musical structure [2]. Moreover, applying the method of principal component analysis to individual performers, they find consistent tendencies of microtiming, groove-like, that differ from inconsistent timing inflections, rubato-like.

[1] In fact, this document is highly "biased" due to our own experiences, knowledge, areas of research, and geographical localization!

Between 2008 and 2011, composers Pablo Cetta and Oscar Pablo di Liscia worked in a research project on the application of Set Theory, Combinatorics and Group Theory to the organization of pitch class sets. Besides several publications [6, 9, 7, 8, 21, 5], the main production (still maintained and usable with free license and open code) is the *pcslib software* [4]. Psclib is a library of external objects for Pure Data, allowing for the application of pitch class sets and combinatorial matrices to analysis and composition [16]. It was initially developed as part of the project "Development of computational applications for the organization of tempered pitch in musical analysis and composition" (2007-2008), Universidad Nacional de las Artes, Buenos Aires; then it continued as part of the project "Musical applications of Sets and Combinatorial Matrices of Chromatic Pitches" (2009-2010), Universidad Nacional de Quilmes, Buenos Aires.

Pablo Amster y Bruno Mesz, mathematicians and musicians, studied the evolution of tango harmony in the period 1910–1960, using complex network methods [15]. Some meaningful tendencies of harmonic discourse that arise from this study are a progressive enrichment of harmonic transitions and a Zipf law for triadic chords, with exponents compatible with a very slow increase of vocabulary in time, according to the aforementioned Zanette interpretation of Zipf's law: the small world characteristics of the networks of harmonic transitions become more definite in the course of time. In a previous article [1] they consider a multi-agent model of emergence of musical styles, based on consensus models and an influence dynamics with delay between the agents (composers); that is, the agents influence each other based not just on their musical styles at a given moment, but also on the basis of their past styles. With this they seek to model delays in cultural propagation in, for instance, the emergence of jazz or tango, due to the slowness of information diffusion in the 19^{th} century. The effect of this delay is to produce more stylistic diversity at the end of the evolution process.

Pablo Riera, a physicist and musician, has studied timbre perception and sound synthesis using mathematical and computational tools. The work "Timbre spaces with sparse autoencoders" presented at the 16^{th} Brazilian Symposium on Computer Music [17] describes a non-linear dimensionality reduction technique to perform timbral analysis of musical pieces and sound synthesis using deep neural networks. Other methods explored for sound synthesis by Riera include the sonification of differential equations. The piece *Ode's ode* (ode to ordinary differential equations) was presented at the Live Coding Music event of the National Institute of Pure and Applied Mathematics (IMPA) [18]. There, he introduced a system for live coding with ordinary differential equations for the synthesis and control of sound, where you can write differential equations formulas and modify their parameters in real time. Riera also participated recently in a work carried out at IMPA, related to the analysis of musical networks. In this work scores of classical musical pieces were transformed into networks whose nodes are musical events. In several of the musical pieces, the networks presented properties of self-similarity or fractal structure [19].

In another line of work, Bruno Mesz, together with the physicists Marcos Trevisan and Mariano Sigman, investigated whether taste words elicited consistent musical representations by asking trained musicians to improvise on the basis of the four canonical taste words: sweet, sour, bitter, and salty. Their results showed that, even in free improvisation, taste words elicited very reliable and consistent musical patterns: "bitter" improvisations are low-pitched and legato, "salty" improvisations are staccato, "sour" improvisations are high-pitched and dissonant, and "sweet" improvisations are consonant, slow, and soft [13]. Projections of the improvisations of taste words to musical space (a vector space defined by relevant musical parameters) revealed that, in musical space, improvisations based on different taste words were nearly orthogonal or opposite. In other publications they design an algorithm to compose "flavorized music" [14], investigate taste-music correspondences of basic music structures (scales and chords) [22] and study the effect

of music in the perception of wine and other drinks [12, 23, 24, 10]. In a more artistic vein, they presented an installation based on [13] at the Cooper Hewitt Smithsonian Museum in New York.[2]

In his M.Sc. thesis, Nahuel Arca considers comparison methods of tuning systems using fuzzy logic.[3] He considers also the case of tuning systems that do not respect octave equivalence.

Jimena Royo Letelier, Chilean physicist and artist, has presented a sound installation at the Henri Poincaré Institute in Paris, associating sonorities to imaginary spaces by solving the wave equation in mathematical spaces such as $\mathbb{R}^3/\mathbb{Z}^3$.[4] This installation works by capturing the vibrations of the building that contains the exhibition and treating them digitally, to generate the sonorities that the building would have if it were embedded in the variety $\mathbb{R}^3/\mathbb{Z}^3$. The sound is reproduced through the vibration of some blackboards. Recently she presented *Muros Invisibles* (*Invisible Walls*) together with her collective artistic group IAKERI. This is an installation that proposes the perception of inequalities between women and men through immersion in a space where matter and sound are revealed and distorted by gender statistics.[5] As a scientist, Royo Letelier works on statistical models and signal processing, with applications to musical emotion detection and characterization of musical genres from audio [20, 3, 11].

In November 2016, the workshop "Recent mathematical and computational applications to music" was held at the University of Buenos Aires, bringing together researchers and students from all over Latin America and prestigious international guests. Organized by Bruno Mesz and Pablo Amster, with funding from the Latin American Center for Interdisciplinary Training (CELFI), it was the first experience of this magnitude at the regional level, which made possible the exchange of a large number of participants, including musicians, mathematicians, physicists and sound engineers, among other careers. The courses covered the following topics:

- *Tuning systems and fuzzy logic*, by Vicente Liern (University of Valencia)
- *Using mathematics and computers in music: a practical guide*, by Dmitri Tymoczko (Princeton University)
- *Introduction to discrete musical systems*, by Shlomo Dubnov (University Of California at San Diego)
- *Mathematical aspects in music*, by Pablo Amster (University Of Buenos Aires) and Bruno Mesz (University Of Tres de Febrero).

The following plenary conferences were also held:

- *Information-Theoretic Music Creativity*, by Shlomo Dubnov
- *Rock Logic*, by Dmitri Tymoczko.
- *Applications for the Theory of Sets of Classes of Heights and Combinatorial Matrices*, by Pablo Di Liscia
- *Analysis of Contemporary Music with Automatic Learning Techniques*, by Pablo Riera
- *Approximations to Language and Music Based on Information Theory*, by Damian Zanette
- *What Happened to the Music when the Spheres were Defaced?*, by Vicente Liern.

In addition, there were communications presented by researchers from various countries and a poster session in which doctoral students from across the region presented their progress. The closing ceremony consisted of a concert with a program linked to the subject of the workshop, including works by various participants and professors D. Tymoczko and S. Dubnov. The presentations and other contents can be found on the workshop page.[6]

[2] https://www.cooperhewitt.org/channel/senses/.
[3] http://cms.dm.uba.ar/academico/carreras/licenciatura/tesis/2015/.
[4] http://esthetopies.ihp.fr/works/text.html.
[5] http://iakeri.fr/en/project_en.html.
[6] http://mate.dm.uba.ar/~tallerdemusica/.

III. Brazil

Although the research on the intersection between music and mathematics in Brazil is relatively recent, it has been considerably developed with a number of ramifications in some research groups and universities of different points of the country, notably MusMat (UFRJ - Rio de Janeiro), NCIS (Unicamp - Campinas), M3 (UFPB - João Pessoa), UFBA (Salvador), and USP (São Paulo).

i. MusMat

The research group MusMat, associated with the Graduate Program of Music of Federal University of Rio de Janeiro (UFRJ), was founded in 2012 by Carlos Almada, Daniel Moreira, Liduino Pitombeira, and Pauxy Gentil-Nunes,[7] aiming mainly at applications of mathematical models to musical analysis and composition. After 2020, three new members – all mathematicians – have joined the group: Hugo Carvalho, Carlos Mathias, and Cecília Saraiva. The Group MusMat edits a biannual journal (*MusMat: Brazilian Journal of Music and Mathematics*[8]), organizes yearly a conference on these subjects, and hosts the first podcast in Brazil entirely dedicated to music and mathematics. In the paragraphs below the research done by current and former members of the MusMat is described, and for more information on other activities as well as historical information about the group, see the last text on this issue.

Carlos Almada pursues since 2011 a research on musical variation, based on Schoenbergian principles of *Grundgestalt* and *developing variation* through incorporation of original concepts, development of new methodological procedures and, specially, computational implementation and formalization, by considering two basic axes: analysis and composition. Concerning the former approach, a number of recent works contributed to the improvement of the analytical model: a study on developing variation applied to serial music [32], a proposal for formalization of metric organization of motivic unities [105], and a deep discussion and reformulation of the theoretical-methodological basis of the research, in a doctorate thesis of one of Almada's students, Desirée Mayr [106]. In 2019 Almada envisaged a new perspective for variation, by considering it under the lens of the Transformational Theory [38, 40], as well as by reinforcing the (not only metaphorical) links with biological variation [42]. These new approaches motivated the author to write a book entitled *Musical Variation: Toward a Transformational Perspective* (not yet published).

Concerning compositional issues, Almada elaborated a system formed by computational programs for algorithmic production of variations from a given basic source of material (*Grundgestalt*, in Schoenbergian terms). This system has been largely expanded in the last years, not only through formal connections with mathematical concepts [31, 35], but also by associations with evolutionary biology [30, 33, 34].

Since 2017, Almada and a team of undergraduate students have been also dedicated to investigate systematically the melodic and harmonic construction of Brazilian popular-music composer Antonio Carlos Jobim. A statistical analysis of a representative corpus of songs by Jobim by a robust computational system especially designed for the task provided a considerable amount of data that has been used for the elaboration of theoretical models, aiming to generalization and further application to other repertoires [36, 98, 37, 39, 41, 43, 107].

Liduino Pitombeira studies formal systems applied to musical composition, including both original and modeled ones. The latter were object of two four-year projects (from 2015 to 2018) in *Systemic Modeling* [142, 136]: in the first project, developed within UFRJ's graduate program in music (PPGM/UFRJ), several *Ponteios* by Camargo Guarnieri were modeled, producing, as

[7] Gentil-Nunes left the group in 2020.

[8] See https://musmat.org/en/musmat-journal/.

a result, compositional systems from which pieces for various instrumental formations were planned and composed [59, 60, 133, 137, 138]; in the second one, developed with advanced undergraduate students, eight short Brazilian pieces for piano or guitar were modeled, and also resulted in analytical papers and new original compositions [134, 135, 139]. The modeling methodology is accomplished through the application of several theoretical frameworks such as: *Developing Variation*, *Partitional Analysis*, *Contour Theory* [132], *Textural Contour*, *Motivic and Gestural Analysis* [140], *Markovian Chains*, [59], *Harmonic Endogeny*, [141], *Linear Partitioning*, *Pitch-Class Similarity*, *Voice-Leading Operations*, *Pitch Class Set Theory*, etc.

Starting in 2019, the present research project, titled *Development of Compositional Systems*, focus on original systems introduced as computational algorithms, graphical plannings, and formal definitions. The development of compositional systems encourages the production of diverse computational libraries and scripts to function as CAC (Computer-Assisted Composition), such as the *CompTools* package developed by Pedro Proença (doctoral student) in *Python*. An intense dialogue with other MusMat projects is also an essential part of this research. Recently, the Ph.D. student Roberto Macedo started compositional experimentation with morphogenetic (dynamic) systems, expanding the field and opening more creative possibilities. Students (graduate and undergraduate), post-doctoral fellows, alumni, and independent researchers connected to this project meet once a week to discuss and expand the theoretical framework, to read books and papers related to the subject, to develop programs, and especially to write papers on compositional systems

Pauxy Gentil-Nunes is responsible for one of the research axes developed by members of the MusMat group regarding the development and applications of *Partitional Analysis* or *Musical Partitional Theory* [81]. The research has produced concepts, methodological and computational tools, as the software applications *Parsemat* and *Partops* (*Partitional Operators*), and publications related to the mediation between musical composition and the *Theory of Integer Partitions*, which is itself a branch of *Number Theory*. Partitional Analysis (PA) has been developed since 2003 by Pauxy Gentil-Nunes and since 2011 by members of the MusMat Group.

The theory is based on the groupings of concurrent musical elements in a musical plot, using criteria arbitrated by the analyst [82]. The first analysis and applications [80] were inspired by Wallace Berry's *Textural Analysis*, but quickly move to applications on other domains, as melodic structure [81], orchestration [109, 90], form [78], analysis of musical gestures [61], and interaction with contour theory [118, 115]. One of the most striking features of the theory is the possibility of perform translations between distinct domains as it always convert the relations and elements to the same general framework.

At the moment, the research is moving to applications closer to musical practice. Moreira [112], for example, studies the stages of implementation of PA in the creative process itself, where the questions posed to the composer imply an amplification of the mappings and concepts. Ramos [143] on the other hand, propose the application of PA in the formalization of the instrumental technique and the partitional analysis of physical aspects of performance (finger, hands) and its relationship with texture.

In theoretical terms, all researchers are dealing, to a greater or lesser extent, with consideration of partitions as a surface element, which receives an organization at hierarchical levels of progressive depth, with roles similar to the Schenkerian levels to pitches. These efforts have led to the concept of *Partitional Complexes* [83, 84] — the coordinated and hierarchical use of a set of partitions, by an entire piece or some section, that ends up expressing at a broader level an implicit partition, called *referential*, that more legitimately reveals the nature of the textural discourse. The concept is being refined and expanded but has already proved useful in clarifying where PA seemed not to work fairly (in the sense of making the structure more visible).

Daniel Moreira is interested in the investigation of musical texture as both an analytical and a compositional tool. As an analytical tool, the first approach was presented in his Masters dissertation, with the formulation of a *textural contour* [118, 115, 113], a proposal that combines the *Theory of Musical Contours* with the *Partitional Analysis* [81, 80]. By using such methodology, Moreira analyzed the melodic contours of bassoon and flute in, respectively, Stravinsky's *Introduction* of *The Rite of Spring*, and Debussy's *Prélude a l'Après midi d'un faune*, to compare with their correspondent textural contours [118, 117]. Moreira also applied this methodology in Mussorgsky's *Promenade* from *Pictures at an Exhibition* and Schoenberg's *Three Pieces for Piano* Op.11 to investigate, respectively, the relation between texture and orchestration and the possible reflects of developing variation techniques in textural domain [116, 114]. Moreover, Moreira implemented two computational applications to facilitate the usage of his methodology (*Contour Analyzer* for general contours [119] and *Jacquard* for textural contour [122][9]), and introduced ornamental functions for texture according to its duration and position within the textural contour [121].

Concerning compositional approaches, Moreira has been formalized compositional strategies for using texture within the creative process [120, 111, 112]. To do so, Moreira has discussed the textural morphology, i.e., the internal components involved in the formation of any textural configuration. From this discussion, Moreira has proposed the formalization of three *textural spaces* to encode the morphology of a given textural configuration. In a general sense, a textural space is defined as the set of textural configurations connected (or related) by a given transformational process. Each space differs from one another in their level of details, using different codes to describe the textural morphology. The most superficial space, called *Textural-class space* (*tc-space*), consists of the simple division of textural components into two abstract structures (line and block), which is related to the general perception of what most people can aurally identify when listening to a piece of music. *Unordered partition space* (*up-space*) is the very proposal of Pauxy Gentil-Nunes' Partitional Analysis [81], in which the textural morphology is portrayed by integer partitions. Finally, *partition layout space* (*pl-space*), the last space, is the most refined description of a textural configuration since it includes the spatial (ordered) organization of its internal components. In his doctoral thesis [112], Moreira formulates a pre-compositional strategy, called *textural design*, in which codes from any of textural spaces are combined into a two-dimensional array ready to be implemented as music[10]. After creating a textural design, the composer may realize it as music, which may be influenced by the articulation of the other musical parameters, as well as the various aspects concerning the compositional practices. In order to discuss this relation between the codes within textural spaces and their realization as music in the score, Moreira defines five *Modes of Textural Realization* that map potential realizations of a given textural code according to different perspectives, such as, pitch content, rhythm, timbre, musical form, and the like. These five modes are not necessarily an exhaustive taxonomy of textural realization, but they cover the most striking features of compositional practices regarding texture.

Carlos Mathias is an associate professor at the Institute of Mathematics and Statistics of the Universidade Federal Fluminense, mathematician, and a professional drummer. His main areas of research are Philosophy of Mathematics Education, Assessment and Curriculum, the dissemination and popularization of Mathematics, and Music and Mathematics. His work in the latter area has two main branches. The first one relies on education, more specifically on the use of rhythm for developing metaphors and images around arithmetical and geometrical concepts for visually impaired students. Mathias has developed a pedagogy on mathematical education called Drummath [102, 103]. The second branch relies on the use of Number Theory and Finite Groups Theory on rhythmic mapping and generation. More specifically, his work

[9]Both are available in `www.musmat.org`

[10]This methodology is based on Morris' *Compositional Design* [124].

relies on building mathematical structures for rhythmic representation and the use of these very structures on computational generation of rhythms. Recent efforts on the latter approach have been recently made in partnership with Carlos Almada, with whom Mathias recently wrote an article proposing an encoding system for drum set's rhythms based on Gödel numbering [104].

The main work of Hugo Carvalho is more directed toward audio signal processing, being his main research area the application of statistical methods to digital audio restoration. In a nutshell, this problem consists on estimating the underlying original signal from a degraded version of it. The process is always performed in a digital computer, and usually there is only a single copy of the corrupted signal, that most commonly comes from the digitalization of recordings in old medias such as gramophone disks and wax cylinders. This lack of observed information makes the inverse problem quite hard to be solved, and therefore, statistical methods may be employed as means to ease this issue. Audio restoration *via* bayesian methods was the main topic on Carvalho's doctorate thesis [54]; some of his publications on the subject on journals and conferences are [52, 53, 48] and specially the more recent [56], where a solution to the problem of restoring audio signals degraded by severe pulses with significant low-frequency content[11] was proposed.

More recently, Carvalho became interested in music information retrieval (MIR), an area within audio signal processing concerned with obtaining high-level information (such as genre, mood, chords, and even the music sheet) from the raw audio signal. Clearly some pre-processing must be made in the time series containing this unstructured data in order to extract more meaningful information and potentially mimic our cognition when interpreting music.

Another of his more recent research interests are the application of probabilistic methods to music composition and analysis. More specifically, in the work [77], with Liduino Pitombeira, the authors employ Markov chains to model the evolution of rhythmic textural partitions and melodic contours of a piece (in this case, the second movement of the *Five Movements for String Quartet, Op. 5* by Webern), and as an example Pitombeira uses the extracted information on the planning and composition of an excerpt of the first movement of his composition *Plate Two: Érebo, Op. 255*. Carvalho also is also working with Carlos Almada in a markovian model applied to chord classes on Tom Jobim's music. Finally, he also wrote a musically-oriented introductory tutorial on Markov chains [55].

Cecília Saraiva is a mathematician and an amateur musician interested in several interactions between the formal language of mathematics and the musical world, specially in the power of algebraic tools to describe and explore musical possibilities, from analysis to composition. Her academic background in Algebra (noncommutative algebra and algebraic geometry) sets the direction of her interests toward the reach of languages such as group and set theory in music. Her main goal at the moment is to study ways in which algebraic tools can unveil musical aspects so that analysis and creation can be mingled. She joined MusMat by the end of 2021 MusMat Conference, after her talk in the round table about algebraic processes and music, in the context of the Twelve-Tone legacy.

In 2020, she advised an undergraduate thesis written by student João Lucas dos Santos Braga studying group theory applied to Neo-Riemannian basic language. João Lucas used the PLR group to analyse excerpts of music created by himself [50].

In 2021, Cecília presented a short talk in an event at UFRJ (Federal University of Rio de Janeiro) called *Celebrating Women in Mathematics*, designed to draw attention to female production and current activities or accomplishments in pure and applied mathematics. The talk *In search of Beauty: algebraic inquiries on the musical fabric* was inspired by questions set in Hermann Weyl's

[11] This problem occurs, for example, when the needle-arm mechanism of reproducing devices passes through very deep scratches or glued breakages.

classic "Symmetry". This talk gave birth to a four-day workshop in 2022 edition of the same event. The workshop *Introduction to Musmathics* primarily focused on presenting the area to mathematics undergraduate students, ended with a basic introduction to neo-Riemannian/transformational theory.

Cecília is currently working with undergraduate students in two projects: one is dedicated to explore some available tools from group and transformational theories in jazz and Brazilian popular music with former student João Lucas S. Braga. A second project, in a more advanced status, will soon be in its final form and uses the language of graph theory to study symmetries and interesting patterns in Brazilian popular music, and also proposes ways to use these tools to teach mathematics in elementary/high school. This work is a project carried out with student Paula Leal.

ii. GPA - UFRJ

The Audio Processing Group (GPA) is part of the Signals, Multimedia and Telecommunication Lab (SMT) at the Federal University of Rio de Janeiro (UFRJ). Officially created in 2000, it is a Brazilian Electrical Engineering research group exclusively dedicated to audio research. Over these years, GPA has produced 10 doctors and more than 30 masters in the field.

Covering as main topics audio restoration [56] and enhancement, spatial sound and acoustic localization [93], audio quality assessment [49], audio coding, and audio analysis and synthesis [58], since its origins the group is characterized by a strong link with Music. In addition to its ubiquity in applications from restoration to spatial sound and coding, Music has always motivated a considerable part of the Group's work, in the form of regular contributions in the field of Music Information Retrieval (MIR) involving alternative time-frequency representations for music [57], pitch tracking in polyphony [47], rhythm analysis [79], separation of musical sources [76], audio-to-score alignment and expressiveness in music, among others.

Current highlights in GPA's production are: the development of combined time-frequency representations for musical signals with improved resolution in both time and frequency aspects [74]; and a set of investigations in the rhythmic analysis of Afro-Latin American musical genres [144]. This last topic has permeated a lasting cooperation with the Uruguayan GPA (Grupo de Procesamiento de Audio), which included a two-year project with Télécom Paris and Centrale Supélec (France). In Brazil, GPA has collaborated with UFRJ, CEFET-RJ, IME, LNCC, UERJ, UFAM, UFSC and Unicamp among teaching and research institutions; and HP, Microsoft and Globo among the companies.

iii. USP

Since the beginning of the current decade, Paulo de Tarso Salles, a recognized expert in structural aspects of Villa-Lobos's music, has been especially interested in studies about symmetry and co-related subjects. Two of his recent papers are special noteworthy in the present review, due to their theoretical-methodological importance: the first one applies Salle's original formulation named *redes de transformações harmônicas* (*harmonic-transformation networks*) in the analysis of some of Villa-Lobos' pieces [146]. The second study addresses the concept of *regiões Euler* (*Euler regions*) which is connected to Richard Cohn's hexatonic and octatonic cicles [147].

Three of Salles's former students act as outstanding collaborators in the expansion of these researches, which mostly involve application of mathematical models for the understanding of Villa-Lobo's creative processes: Ciro Visconti investigates symmetric relations in Villa-Lobos' guitar *études* [153], and proposes an interesting spacial approach, named *axes of contextual inversion*, for

bringing up tonal relations in selected passages of the Post-tonal repertoire [154]. Walter Nery Filho works with intricate geometric representation of Neo-Riemannian operations [125]. Joel Albuquerque's recent PhD thesis [29], grounded in the Group and Transformational theories, presents an exhaustive typology of spatial schemes that intends to map symmetric connections between sets in Post-tonal music. Along the last years, Albuquerque has also been published some related papers in co-autorship with Salles and other colleagues [29, 28, 26].

Rodolfo Coelho de Souza, who hosts a degree in Engineering, coordinates two basic lines of research that dialogue with both music and mathematics. The first line addresses physical-mathematical modelings of sounding aspects of Brazilian native musical instruments, associated with algorithmic strategies, and involving methods of wave-terrain synthesis, additive synthesis with noise, among others, being also articulated with the fields of algorithmic composition and artificial intelligence [72, 73, 131, 66, 65, 62]. Simultaneously, Coelho de Souza has been developed several topics associated with pitch-class set theory, proposing original and particular views, aiming both to analytical and compositional applications. His main contribution for the expansion of this field is the formulation of a special theory addressing non-ordered-intervals classes, besides studies concerning intrinsic properties of some special pitch collections, as well as neo-Riemannian mappings [71, 69, 68, 70, 67, 64, 63, 152].

iv. NICS - Unicamp

The Interdisciplinary Nucleus of Sounding Communication (Núcleo Interdisciplinar de Comunicação Sonora – NICS[12]), created on April 5, 1983, focuses its research interests on the different manifestations of sound, under the perspectives of information, cognition and creativity. NICS brings together researchers from different fields, particularly arts and the sciences, and develops interdisciplinary projects that aim to broaden the relationship between musical conception and the proposals of new production, control and sonic analysis models. Since 1994, the Nucleus has focused on Mathematical Modeling, Computational Simulation, Musical Composition, and Computational Music [151, 108, 130, 145, 99, 100].

The NICS has established agreements with several national and international institutions. Among them, we highlight a partnership with the Institute of Neuroinformatics (INI) of the Swiss Federal Institute of Technology (ETHZ), in Zurich, in the production of the ROBOSER system, and VOX POPULI, an evolutionary system applied to algorithmic musical composition developed by researchers Jônatas Manzolli (NICS), Artemis Moroni (Division of Robotics and Computational Vision - DRVC of CTI Renato Archer, from Campinas), Fernando Von Zuben, and Ricardo Gudwin (FEEC); this system was awarded the Dream Centenary Computer Graphics Grand Prix 99 in Aizu, Japan, in the category of Best Interactive Installation. The current research lines are (1) Musical Analysis with Computational Support, focused on the computational support for musical analysis as a way for understanding the sound dimensions of music; (2) Multimodal Creation, as research on creative processes from the reflection on the interaction of the musical gesture, human-machine interaction, computational models applied to sound design and interaction with other languages in multimodal performances, and (3) Multimodal Interaction in Therapeutic Procedures, through applications of interactive musical and multimodal technology anchored in computational models and interdisciplinary studying, covering musical and neuroscience fields.

[12] https://www.nics.unicamp.br/

v. Genos Research Group

In Bahia, the relationship of music and mathematics are mainly associated to the Genos research group,[13] a multidisciplinary group of research of theory, composition, and musical computation linked to Federal University of Bahia (UFBA). Genos has developed research projects in four main areas: (i) codification, (ii) music algorithmics, (iii) Pitch-class set theory, and (iv) analysis supported by statistical data.

Pedro Kroeger, leader of Genos, has developed a research on codification of musical structures in a higher level than the traditional codification of pitches and rhythms through numbers and letters in such a way to express wider structures as all the fugue's subjects of Bach's well-tempered clavier or chords of a harmonic progression, considering the representation of chordal roots, harmonic rhythm, chordal qualitites, and their possible inversions, among others [97]. In his doctorate thesis, Kroeger developed a meta-language for sound synthesis [94], and he also explores automated harmonic analysis [95] and statistical analysis addressing occurrence of consecutive fifths and octaves, cadences, voices, augmented-sixth chords, voice leading and crossing, seventh resolution, and final cadences in Bach's 366 choirs [96].

Jamary Oliveira developed compositional applications such as *MUSICOMP* (1981), that prints a serial matrix, *SONG.DATA* (1985), which provides an analysis of melodies [127], and *Pitch-Class Processor* (PCN[14] – 1992), that consists of a pitch-class calculator for analysis based on pitch-class set theory. Oliveira has been also interested in pitch and rhythm codification [128] and twelve-tone composition systematization [126]. The MusMat Conference of 2020 was in homage to Jamary, who left us this same year.

Marcos Sampaio is, perhaps, the main researcher on the theory of *musical contour* in Brazil [101, 124, 123]. His doctoral thesis presents a complete review on contour theory, pointing out some problems and inconsistencies in some algorithms proposed by different authors [150]. Most of these concepts were implemented in the software called *MusiContour*.[15] Sampaio is also responsible to the refinement of the reduction [149] and similarity algorithm, proposing two new algorithms to enable the comparison of contours with different cardinalities [148].

Other former students related to Genos are Dennis Queiroz Carvalho, whose interests lies on axial symmetry of some pitch-class sets, like the set-class [8-17] [51], and Natanael de Souza Ourives, who is interested in reviewing combinatoriality of twelve-tone structures [129].

vi. Mus^3 Research Group

The Mus^3 group is linked to the Federal University of Paraíba (UFPB) and to the NICS - Unicamp (São Paulo) and develops researches in the areas of Musicology, Sonology and Music Computing. The lines developed are focused on the formalization of the timbre-related composing dimensions at their conceptual level, covering the research of Didier Guigue [85, 87, 88] and *Textural Planning*, referring to the work of José Orlando Alves [45, 46, 91, 92, 75, 110].

Estética da Sonoridade (*Esthétique de la Sonorité*) is the title of Didier Guigue's referential book [86, 87], which constitutes one of the essential publications in musical analysis in the country. The theory is constructed from the concept of *Composite Sound Unit* (*CSU*), defined as the combination and interaction of musical *primary* components (a collection of pitches) with *secondary* components – namely intensities, ranges, registering, densities, modalities of statistical distribution of pitches or other low-level elements, e.g. deviations, entropy and others. This theory led to the development of

[13] https://genosmus.com/

[14] See http://www.angelfire.com/music2/bahia/pcn/pcn2001ptb.htm

[15] See https://pypi.org/project/MusiContour/

a library of functions in the OpenMusic environment, the *SOAL* (*Sonic Object Analysis Library*) [89]. There is an ongoing collaboration between the groups Mus[3] and MusMat around the research on musical texture, generating sharing of concepts and tools in publications and software [90].

Textural Planning is developed by José Orlando Alves [44, 45, 46] and aimed at applications in the compositional process. Its starting point is the use of matrices to define modes of organization of durations and texture, from Wallace Berry's theory (1976, see [44]). The work has resulted in publications and papers, both by Alves and his advisers, as well as a software (*TexturalCalc*) developed by Felipe Grisi [92] in Java, which calculates the level of textural complexity of each instrumental configuration of the work, based on the number of real components and their relation to density-number.

IV. Colombia

In the context of Latin American systematic musicology, Colombian contributions are paramount for a clearer understanding of *subjective categorization, gesture theory, topos theory, homotopy theory* and *homology theory* applied to music, as well as computational developments for music composition and analysis. For a historical introduction on the latter, Cuéllar-Camargo's article [159] provides a first approach that obviously needs to be actualized. In respect to subjective categorization of music, with a great regional, potential interest, we find Pablo Mendoza-Halliday's PhD dissertation [162], *Musical genre as a dynamic process: a semio-cognitive theory of the categorization of music*, in short, an effort to explain to musicians how social use of music requires fuzziness criteria analogous to semantics and pragmatics in language, in order to produce "musical sense". This conceptualization involves complex dynamics for building and distinguishing musical genre and generic indications, being a novelty in Latin American literature:

> *Musical genre* is a concept widely used to name "kinds of music". However, in music theory this concept has provoked debates on how it is conceptualized, what is categorized, how is such categorization achieved, what is its function, and if it is necessary, or if it can be avoided or replaced. Regardless of the posture on these subjects, categorization of sensitive experiences such as music is cognitively inevitable. It is one of the ways to make knowledge apprehensible. For the same reason, the categorization of music is one more tool in the construction of the meaning that music acquires and that allows to make use of it.
>
> The aim of this thesis is to review the concept of musical genre through a theoretical proposal that reassesses the current paradigms of categorization and that, from a viewpoint of its socio-psychological dimension, explains how the category is comprised, how it can be analyzed and how it affects the processes of musical signification through generic indications. The cognitive (enactivist) and semiotic (Peircean) theories of categorization are integrated in this proposal in order to conceive genre as a dynamic process that includes phases of cognitive categorization, conventionalization, and taxonomization. This allows us to have a broader approach to this form of categorization of music, and it can contribute to rethink the role of genre in music theory. Furthermore, analyzing genre and generic indication as signs of music leads to an understanding of its function as hermeneutic guides to music. This can shed light on how to appropriate this semiotic strategy and optimize its use.

Whether Mendoza-Halliday does not develop mathematical formalization or computational direct applications, his logical schematization in fact paves the way for further research in

music and mathematics involving *category theory, algebraic topology* and *fuzzy set theory* with clear computational implications. Furthermore, another Colombian young scholar, Juan Sebastián Arias-Valero [155, 156, 157, 158] seems to fulfill the necessary complement for this task; so the harmonization of Mendoza-Halliday and Arias-Valero scientific goals would be a relevant next step within this domain. By now, Arias-Valero extended Mazzola's *topos theory* [158], both in philosophical and pragmatic perspectives for music theory, contributing to the Peircean and Grothendieckian expansions in musicology (exploring the conceptual fields of *hypergestures, gestures on locales, localic topoi, sites, morphisms of sites, presheaves* [i.e. sheaves on the trivial topology] and *sheaves*). The successful adaptation by Arias-Valero from Peirce, Grothendieck and Mazzola for a mathematical theory of music was encouraged by Valero's former supervisor, Colombian mathematician and philosopher Fernando Zalamea (Bogotá, 1959).

In recent years, Professor Zalamea published articles and books on the analogic deep relationship between *musical creativity* and *mathematical creativity*, with particular attention on concepts from Charles S. Peirce and Alexander Grothendieck, from a specific philosophical point of view:

> In fact, Peirce, Einstein and Grothendieck involve, with different techniques, the observer's framework and the partial cognitive dynamics of the agents. In particular, in the Grothendickean *making*, a network of incessant *transfers, translations, translations* between apparently distant regions of mathematics is introduced." (Zalamea, 2014 [165, p. 112]).
>
> In categories with some good properties of compositionality and coverage, an abstract topology (Grothendieck's topology) can be defined by (sub)collections of morphisms that "tie" well each other. The categories of pre-sheaves (categories of functors to values in the category of sets) verify these good properties of compositionality and coverage, and abstract topologies can be defined there. Grothendieck's topos come from categories of pre-sheaves that are "situated" around a given abstract topology (those categorical environments are also called then *sites*). *A geometry of the situation precedes logic*, against many postulates of analytic philosophy. (*loc. cit.*, footnote).

This theoretical stressing on how the "observer's framework" is related to "the partial cognitive dynamics of the agents" is an issue immediately associated to the self-referential complexity of music (as referred in Pareyon, 2011 [163], both in terms of musical *poietics* and *poiesis*). Furhermore, Zalamea's synthesis and clear explanation on Grothendickian categories, makes also clear how urgent is for regional musicology, building ties between Mendoza-Halliday and Arias-Valero's prospects. Moreover, the Peirce-Grothendiek adaptations to music theory and practice, unveils a shortcut for communication between Mesoamerican aesthetics (and more specifically from Tepon-azcuauhtla interpreted as a *sheaf*, in its Grothendiek-topologic sense) and modern mathematics in dialogue music. Finally, this topological conception of music can be seen as an analogy to the "mathematical *pointless* thinking" as explained by Đurđevich in terms of *quantum geometry* (see: Mexico and Central America, below; notice that the musical theory of Zalamea is also enriched by musicologist and composer J.S. Lach-Lau, mentioned in the following Section, too).

Recently, Colombian post-graduate researcher Nicolás Jaramillo-Ramírez (Julio Estrada's former student) successfully defended one of the best master's theses at the Faculty of Music - UNAM, in the area of composition [160]. This thesis, with the title "Infinite Points between Yes and No: Percepts in Musical Creation", explains Boolean logic with extensions to infinite-valent logic applied to musical composition, Fuzzy Set theory, and the Continuum-Discontinuum theory (based on Estrada's doctoral thesis). Jaramillo-Ramírez's dissertation paves the way to better understanding the link between Peirce's semiotic logic (*ponecipuum*, *percipuum* and *antecipuum*), towards category theory and its possibilities for a logic of intersemiosis and the intersemiotic

continuum (as postulated by Pareyon, 2011 [163, pp. 108–114]).

V. Mexico and Central America

Regarding the intersection of music and mathematics, the cultural dynamics of Mexico is one of the most complex in the region. This is due both to the cultural breadth of the so-called Greater Mexico (which includes the regional fringe that extends between California and Texas,[16] as well as Central America), as well as to the depth of its ethnic and cultural history. However, this section is limited to what Lluis-Puebla ([223, p. xxv]) defines as "modern mathematical theory of music from the 1980's". As a specialized academic issue, this activity is mainly centralized in six institutional clusters: (i) FaM/PMDM (UNAM), (ii) IIMAS/ICAT/FCiencias (also UNAM), (iii) Cenidim/CMult/ESM (CENART), (iv) FaM (UV-Xalapa), (v) CR/CMMAS/ENES (UNAM-Morelia), (vi) CUCEI/CuCosta/CUCBA (UDG). This section ending does explain how students and scholars from Guatemala and El Salvador participate on this network. Additionally, an appendix on Costa Rican specialists is provided.

i. FaM/PMDM

The Faculty of Music (FaM) and its Postgraduate Program in Music (PMDM) of UNAM (National Autonomous University of Mexico), historically leads the study and development of music and mathematics in Mexico. Within this framework, and for the last 25 years, as mentioned at the end of the first text on this issue, Julio Estrada (emeritus professor, Faculty of Music, UNAM, Mexico City, and IIE-UNAM researcher), adopted and adapted the heritage of Carrillo and Nancarrow as "cataracts and trees" of harmony in terms of space and time (see [191, p. 101]), through a variety of approaches between what Estrada himself defines as *discontinuum* and *continuum* [200]. However, a first attempt to reformulate music theory, particularly from its symmetries associated to a musical algebra and syntax, was introduced in the context of Boolean algebra and finite group theory; as Estrada explains [199, p. 159], in the sense of an analogic reasoning:

> Finite Group Theory operations can be described as logical transformations that arise when working with the elements of an abstract language which, when applied to other languages, prove to be of great interest and usefulness as they show some operative mechanisms that man practices throughout his physical and intellectual action.
>
> This theory studies the symmetries of forms. When applied to music, it shows the coincidence with those symmetries that have been traditionally used (retrograde, inversion, and retrograde inversion) and other symmetries that would pass unnoticed with regular systems of musical analysis.

Along most of his theoretical development —with very specific applications to musical analysis and composition in multilayered criteria for distance and scalar production— Estrada's starting point is a matrix design based on the symmetry of scales and their permutation/combinatoriality as pitch collection. As a matter of fact, one may trace this strategy through his first steps for modeling scales in terms of symmetric arrangements, partitions and *logical transformations* (see, e.g.: [199, pp. 167–168], [200, pp. 171–186]; with systematized employment in Estrada, 2001 [201],

[16] For an introduction to the problematic conceptualization of mathematical understanding in communities along this fringe, see Gutstein *et al.*, 1997 [210, p. 731], in parallel to the understanding of music theory, since mathematics and music theory "is usually seen as a neutral and objective subject, devoid of specific class, cultural, or political content; the connection to social activism may seem to be a stretch. Even when mathematics [and music theory] learning is seen as personally empowering, it is rarely thought of as having broader, social justice implications for those learning it".

2002 [202], 2006 [203]). When adapting these *symmetric principia* to independence of interval size/distance (a concept closely related, both to Carrillan aesthetics, and to the distinction between *discontinuum* and *continuum*), an infinite yet *well-formed* relationship is foreseen across pitch, scale, texture, timbre, rhythm and meter, with "great interest and usefulness" for creating, understanding and transforming music. Estrada also produced important research to connect the technical view of his abstract methods, with "cultural integration", avoiding the temptation of put rationalism forward social and historical values of regional culture (see Estrada, 2011 [204]).

Among Estrada's students and collaborators are Leticia Cuen [196, 197], mainly focused on the structural role of the Golden section (φ) in modern music; Mauricio Rodríguez [245, 246], occupied in the study of the *algorithmic expressiveness* in scores based on signal analysis and graphical transformations, and how such *expressiveness* is related to rhythmic and metrical complexity; and Víctor Adán [169, 170, 171, 172], post-graduated from The MIT, and specialized in the analysis, modeling and resynthesis of music from its hierarchical structures interpreted from a dynamical systems and signal processing approach, as well as interested in the concept of *control* in the context of musical symbolization and programming for scores production.

Adán collaboration with Estrada is particularly meaningful because of the implementation of Estrada's $d1$ *theory* and harmonic permutahedron, where $d1$ refers to any harmonic or interval distance (d) which minimal constructive and/or syntactic order equals 1. Given that, in practice, 1 may be defined as any "basic" interval, the $d1$ *theory* may be useful in any periodic lattice *kind of* structural harmony, and may also be implemented for any periodic structure of time. Its corresponding software allows the user to experiment, for example, the permutation of a series of time intervals in an oscillation with constant amplitude, or the representation (and sounding output) of different slopes of curves product of the modification of a *continuum/discontinuum* variable.

Another leading scholar historically active at FaM/UNAM is pianist, flutist, teacher, composer, mathematician and programmer Roberto Morales-Manzanares [236], later a full-time Professor at the Faculty of Music, University of Guanajuato, where he founded the LIM (Laboratorio de Informática Musical/Musical Computer Lab). During years 1980's and 90's, Morales-Manzanares settled the computational grounds for ESCAMOL, an algorithmic program for music composition and visual-spatial interaction, actually employed by the University of Berkeley, San Jose State University, Yale University and McGill University. Morales-Manzanares' post-graduate students include J.R. Cabezas-Hernández [190], Edmar Soria [248, 249], Mauro Herrera-Machuca [212], and J.E. Gómez-Elizondo [207], all of them occupied on audiovisual and "transmedia hybrid creation" employing mathematical models, mostly implemented in SuperCollider and ESCAMOL, and extensions to develop strategies of music and meta-music creation.

ii. IIMAS/ICAT/FCiencias

The UNAM Faculy of Sciences (FCiencias) located at Ciudad Universitaria (Mexico City's south) has a meaningful tradition crossing over music, mathematics, physics and other fields in natural sciences, and for academic and research in musical projects frequently collaborates with the Institute of Research in Applied Mathematics and Systems (IIMAS) and the Institute of Applied Sciences and Technology (ICAT).

An internationally recognized pianist, Emilio Lluis-Puebla is also —since 1980— Algebra Professor at the Department of Mathematics (FCiencias, UNAM), where he formed at least three generations of scholars within both domains of music and mathematics. His research and teaching expertise includes Theory of Homotopy, K-Algebraic Theory, Vectorial Sheaves, Cohomological Operations, Infinite Loop Spaces, Homological Algebra, and Groups Cohomology

(see [218, 219, 220]). His theoretical approach to music and musicology was firstly inspired by Birkhoff's (1933) *aesthetic measure* [188], Borel & Serre's (1958) view on the Riemann–Roch theorem [189], and Lendvai's (1979) geometric conceptualization of musical form analysis [217].

Since 2004 Lluis-Puebla closely collaborates with Guerino Mazzola and produced a wide number of articles, books and academic seminars and conferences on music and mathematics [221, 222, 194, 240]. Among Lluis-Puebla's most internationally renowned post-graduate students are Flor Aceff (see [167, 168]), Jesús David Gómez-Téllez (see [208, 209]), Mariana Montiel, and Octavio Agustín-Aquino. Montiel adds to her prolific research output, the merit of probably being the first female scholar in Latin America being completely devoted to the study of a number of facets linking music and mathematics with algebraic, geometric, combinatorial and topological methods, and pedagogic, psychological and other applied approaches "to understanding musical phenomena" (see [229, 230, 231, 232, 233, 234, 235]). Agustín-Aquino, in his turn, produced in the region the widest specialized literature on the mathematical theory of musical counterpoint (see [173, 174, 180]), including $2k$-tone equal temperament [176], microtonal counterpoint extensions [177], extended counterpoint symmetries and continuous counterpoint [179], as well as other computational counterpoint worlds [178] (see also Nieto-Ramos [237]).

Pablo Padilla-Longoria, Professor at UNAM's Department of Mathematics, and at IIMAS, granted his Ph.D. at CIMS, New York University; he also graduated in piano from Mannes College of Music, New York, and is an active performer of organ and harpsichord. He created the Seminar on Music, Computation and Mathematics (SEMIMUTICAS – IIMAS), and developed pioneering research, in collaboration with Prof. Alejandro Ramos-Amézquita, employing mathematical modeling for the study of Mesoamerican and New Spain buildings' archaeoacoustics. During 2016-2017, Padilla-Longoria was a visiting member of the Fitzwilliam College of the University of Cambridge, where a research project on mathematics and music was carried out in collaboration with F. Knights (Fitzwilliam College) and D. Tidhar (Center for Music and Science) (see [238]). His students include Chávez-Zamorate [195], J.R. Cabezas-Hernández [190], J.A. Lobato-Cardoso [224, 225], and Iván Paz [244], most of them programmers/analysts, familiar to cellular automata, neural networks and AI algorithms applied to musical performance and composition.

Felipe Orduña-Bustamante, is researcher of the Acoustics and Vibrations Group, ICAT-UNAM. He has a degree in Physics (UNAM, 1987), a Master's degree and PhD in Sound and Vibrations (University of Southampton, England, 1990 and 1995). He works on instrumentation and acoustic measurements, digital signal processing, musical acoustics and technology, and computational musical retrieval (Bañuelos & Orduña-Bustamante, [187]), and he also contributed to the acoustic investigation of musical instruments Native to Mexico, studied through physical, computational and mathematical methods (Herrera-Castro et al., 2018 [211]; for other technological and systematic applications on the same topic, see also García-Munoz [206], and Pareyon & Pina-Romero [241]).

Micho Đurđevich, researcher at the Institute of Mathematics (FCiencias), UNAM, is a specialist on C-Algebras, Galois theory, principal quantum sheaves, and quantum geometry, mainly focused on quantum circles and quantum spheres. This mathematical repertoire also includes Cohen forcing within the context of Grothendieck and Giraud theory of Topos (developed by Mazzola [161], and emphatically applied by Lach-Lau [214] to musical composition; an approach also used by Gómez-Téllez *et al.*, 2017 [209]). Đurđevich output [198] also facilitates an accurate mathematical context for the understanding of Teponazcuauhtla, the non-linear set of musical aesthetics described in the first text on this issue, especially with the use and conceptualization of the teponaztli conceived as an experimental (theoretical but mostly practical) device for quantum circles, where "the entire fabric of space is considered as the one and indivisible whole" (Đurđevich, *op. cit.: Introduction*; completely empathetic with the historical view in the first pages of the present study).

iii. Cenidim/CMult/ESM

The National Center for the Arts (CENART) in Mexico City, also constitutes an important cluster for musical research and interdisciplinary production. CENART is the home of the National Center for Music Research, Documentation and Information (Cenidim), the Multimedia Center (CMult) and the Escuela Superior de Música (ESM) that depends of the National Institute of Fine Arts (INBA). In fact, engineer Raúl Pavón developed at INBA, in the early 1960's, the first music synthesizer completely made in Mexico, and he also published in Cenidim the first Mexican book on music and computers (Pavón, [242]). The first years of the CENART building complex benefitted from the skills of José Antonio Amozurrutia (Mexico City, 1950), pianist, composer, mathematician and programmer who designed the first CENART intranet and computer clusters at CMult (see: Amozurrutia, 1997 [181]), before starting his pioneering project for applying music theory and computer science to the investigation of genetic epistemology. Later, the first decade of the 21st century was particularly fruitful at CMult, with mathematical/programming seminars and live coding concerts with a celebrated participation of computer scientists and music creators such as Ernesto Romero, Sergio Luque, Iván Paz [244] and Jaime Lobato-Cardoso [224, 225], implementing mechanic, algorithmic and multimedia devices with local developments.

In 2012 Gabriel Pareyon founded at Cenidim the first national Seminar of Science and Music Theories (originally SeCiTem), in cooperation with SEMIMUTICAS and the FaM-UNAM, with students of Pablo Padilla-Longoria and Emilio Lluis-Puebla. This organization was the ground for launching the International Congress on Music and Mathematics (Puerto Vallarta, 2014), with a homage to Julián Carrillo and Alexander Grothendieck synthesized in the book The Musical-Mathematical Mind (see: Pareyon *et al.*, 2017 [240]). Then in 2015 pianist Gabriela Pérez Acosta entered the Cenidim team after completing a neurological study on musical cognition (see: [243]), and she continued organizing SeCiTem, thereafter under the simplified name of Seminar of Science and Music, ever within the cooperation Cenidim-INBA/FaM-UNAM, and which is the most regular forum in Mexico for the discussion on topics bonding music and mathematics. Up to this point, the Seminar monthly coordinates lectures about pitch-class set theory, intervallic theory, microtonal tuning, harmony and counterpoint, music and computational experimentation, mathematical modeling of voice and instrumental spectra, musical syntax and semantics, musical form analysis, music perception and psychoacoustics, and other similar subjects.

iv. FaM/UV-Xalapa

Central-eastern Mexico's activity on music and mathematics is concentrated in Xalapa, the capital city of Veracruz state, mainly within the regular courses, seminars and concerts leaded by Professor Emil Awad [184, 185], graduated *magna cum laude* from Juilliard School and Manhattan School of Music, and PhD in Music from Harvard University. In words of Hebert Vázquez [250, p. 10, 465], Awad is a key scholar in modern history of regional music and mathematics from a North American perspective:

> Despite the fact that in the last 30 years [before 2006] the theory of atonal music has produced a huge stock of theoretical-analytical literature and has been firmly installed in the curricula of professional music teaching, especially in the Anglo-Saxon countries, in Mexico it is still practically unknown. [250, p. 10] In this regard, it is necessary to highlight the important academic work that composer Emil Awad has been doing since the 1990s; pioneer in Mexico, both in the field of teaching the theory of atonal music and in Schenkerian analysis. [250, p. 465]

Hebert Vázquez himself is author of the most thorough compendium on atonal music theory ever published in Mexico (see: [250]), both for analytic and compositional interest. Altogether with Vázquez, other Mexican composers of his generation that perceive their musical output rather as demonstrations of their mathematical postulates are Antonio Russek (1954–), Ana Lara (1959–), Víctor Rasgado (1959–), Juan Fernando Durán (1961–), Juan Trigos (1965–), Armando Luna Ponce (1964–2015) and Georgina Derbez (1968–), mostly active in Mexico City.

v. CR/CMMAS/ENES-UNAM-Morelia

Morelia city in Central-Western Mexico is the home of the oldest public music school in the country, the Conservatorio de las Rosas (founded 1743), which have had prestigious teachers in music composition and analysis, including Juan Sebastián Lach-Lau [213, 214, 215], a scholar that reunites to his musical abilities the domains of algebraic topology, sheaves and fibre bundles, connecting concepts and methods borrowed from Badiou [186] and Grothendieck (see: Zalamea, [251, 252]). As Lach-Lau explains [216]:

> The above investigation is inscribed in a larger project that seeks to develop a model based on ideas stemming from mathematical phenomenology (close to the idea of a *topos* by Grothendieck), from which the conditions of appearance of musical events and structures are stipulated and fixed in terms of perceptual and conceptual limits and gradations, derived from logical, geometrical and topological objects. What is sought is that the expressive range and the ways of setting these ideas in a musical context covers a wide aesthetic ground. The model invites the positing, at several time scales and in interaction with the *continuous* (morphological), as well as the *discrete* (structural) facets of musical form, their conditions of appearance, both real and ideal: on the one hand there is a conceptual aspect, extensible to all kinds of *forms* and abstract relations; on the other hand, there is the empirical feature, comprising selection and generation of *materials* that exert resistance: found objects, instrumental modes of playing, sonic experimentation, improvisation, etc. This conjunction lays the ground for a synthetic approach to composition, involving both intuition and rationality. It does not impose any kind of a-priori aesthetic and it is not reducible to a method or technique, but is a way of setting out abstract sonic spaces and their transformations in a framework of structural relations and operations that can guide and assist the compositional act.

In fact, Lach-Lau's research and output provides a clear and interesting network for a virtuous connection of theories formulated by Colombian scholar Fernando Zalamea (above mentioned), not only over Grothendieckian mathematics, but also importantly over Charles S. Peirce, as a key author for better understanding the link between non-linear algebras and regional historic interpretations of music (Teponazcuauhtla's extensions on Carrillo, Nancarrow, Estrada, to which we may add the quantum spaces and trajectories studied by Đurđevich, [198]). Lach-Lau's valuable work on music and mathematics pedagogy also impacts on CMMAS (Centro Mexicano para la Música y las Artes Sonoras) and the ENES-UNAM, where he also teaches.

vi. CUCEI/CuCosta/CUCBA (UDG)

Even when the University of Guadalajara (UDG) does not develop any musical research program in music and mathematics, in 2014 the CUCEI (University Center for Exact Sciences and Engineering) and the CuCosta (University Center for Jalisco State Coast) united forces to celebrate the International Congress on Music and Mathematics in Puerto Vallarta, already mentioned, with

Pareyon as a main organizer, and also with valuable support by E. Lluis-Puebla, S. Pina-Romero, O. Agustín-Aquino, J.S. Lach-Lau, R. Morales-Manzanares and P. Padilla-Longoria. Sharing this enthusiastical spirit, CUCEI's Department of Computation acquired an anechoic chamber and opened a series of seminars on acoustics, psychoacoustics, electronic music, music & sound analysis, and selected topics on related mathematics. Pareyon students at CUCEI included Emilio Ceja-Cárdenas and Rodrigo Castro López-Vaal (actually PhD student at the Postgraduate School of Music, Georgia State University, Atlanta, under de guidance of Mariana Montiel). A more recent student of Pareyon, both in UDG and FaM/PMDM, is Axel Avendaño (Guatemala City, 1980), devoted to construction and re-construction of a variety of aerophones and to explore their relationship with Mayan languages actually spoken in Central America, employing Fourier analysis and syntax modeling of speech in a tonal, Native language (K'iche') retrieving musical harmony, meter, rhythm and texture information (see: Avendaño, [183]). In recent years Pareyon permanently collaborates with Martha Georgina Orozco-Medina (from CUCBA, University Center for Biological and Farming Sciences), within her Seminar for Urban Noise Research. A fourth student of Pareyon, graduated as seismologist from CuCosta (Puerto Vallarta), is environmentalist and composer Yair López [226] who actually leads there a scientific program for sounding landscape register and ecological interpretation of music employing mathematical and computational tools.

vii. Costa Rica & El Salvador

Although Costa Rica has a long-range tradition of musical and mathematical communication with Mexico (e.g. through the education and career development of composer Rocío Sanz Quiroz (1933–1993), a former member of the Liga de Compositores de México who employed her own serial methods), the country has a noticeable independence for music research and production. For example with the output of composers Bernal Flores (1937–) whom combinatorial strategies for serial composition and mathematical modeling of popular rhythms continued to a following extent in the work of Mario Alfagüell (1948–), who composed a huge number of musical works and taught composition using his own system of numerical series (notice that a recent chapter on mathematical formalization of Latin American dance tradition, as already started by Miranda-Medina & Tro [228], and Rodríguez *et al.* [247], would need, however, an independent synthesis). Younger Costa Rican scholars include Mauricio Soto, graduated as Computer Scientist and Music Teacher at the University of Costa Rica, and M.Sc. in Software Engineering at Carnegie Mellon University, as well as Gabriela Chavarria-Soley [193], who works on mathematical modeling of absolute pitch hearing, in the Universidad de Costa Rica. A former student of Roberto Morales-Manzanares at the PMDM-UNAM, already mentioned J.R. Cabezas-Hernández [190], is also a distinguished Salvadorean scholar making part of a new generation of Central American specialists interested on topics connecting music, mathematics and computer science.

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